Domain of the Convertible Rotor

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The convertible rotor (CR) is a means that provides lift in the hover mode and propulsion in the cruise mode of a VTOL aircraft. In the wide disk loading spectrum of this family of aircraft, ranging from 2–2000 psf, various aspects are examined, especially that of noise. The need for a "balanced VTOL solution" is demonstrated. Such various solutions are feasible depending on operational requirements, namely, the helicopter rotor (for low cruise speed and prolonged hover) and the fan lift engine (for supersonic cruise speed). The convertible rotor with a disk loading of approximately 25 psf is shown to be the best solution when economy at high subsonic cruise speed is required. This loading is also near the maximum that can be tolerated on account of noise, for operation in builtup areas. The aerodynamic mechanism of the convertible rotor is explained and reasons for its high efficiency given. Operating costs, together with journey times between city centers, of a typical convertible rotor VTOL aircraft are given in comparison with those of helicopters and fixed-wing transport aircraft. It is concluded that, subject to technical problems as yet unresolved, the convertible rotor can win an important domain in the field of air transportation.

Introduction

Many papers have already been written on the subject of high-speed VTOL aircraft and some specifically on aircraft with convertible rotors. The theoretical aspect of this field has been extensively explored, and there have also been many experimental investigations with wind-tunnel models and, indeed, with full-scale research aircraft. A stage has now been reached in the technological development of this new branch of aviation where the basic issues involved are generally well understood.

In essentially preliminary explorations however we have reached certain tentative conclusions which in some respects appear to differ from prevalent opinion as discernible in the published literature, and therefore it may be of some benefit to record the background and the reasoning that has led to these conclusions.

The Two Modes of VTOL Flight

VTOL aircraft are essentially hybrids, inasmuch as they are sustained in two heterogeneous modes of flight, namely 1) in hovering, where the predominant air flow is induced from the vertical thrust, and 2) in cruising, where the predominant air flow is due to translation of the aircraft.

In vertical flight, all VTOL aircraft obtain their lift by means of one or more actuator disks. They may be large and open as in the helicopter, or small and buried, as in the fan- or jet-lift VTOL aircraft. The most important parameter of the actuator disk, the disk loading, varies from 2 to over 2000 psf. In forward flight, lift is best obtained by a fixed-wing, and here the relevant parameter is span loading.

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In changing from one mode of flight to the other, a so-called transition is made when both means of support function in a somewhat distorted fashion and at a diminished efficiency. These two modes of VTOL flight differ from one another not only with respect to the means of support but also with regard to control, stability, and in various other important aspects. A VTOL aircraft must, therefore, be fitted with means adaptable to suit both modes of flight, or such means must be supplied in two versions, one for each mode of flight. It follows from this that performance penalties are incurred in both modes of flight. They take various forms but can generally be expressed as parasitic weights. To reduce weight penalties to a minimum is the object of skilful design, in which case a satisfactory "balanced" VTOL aircraft is achieved.

Disk Loading Parameter

The influence of disk loading on the performance of the VTOL aircraft is well known. Rate of change of momentum is equated to thrust which arises either from a small change of speed in a large mass flow (low disk loading) or a large change of speed in a small mass flow (high disk loading).

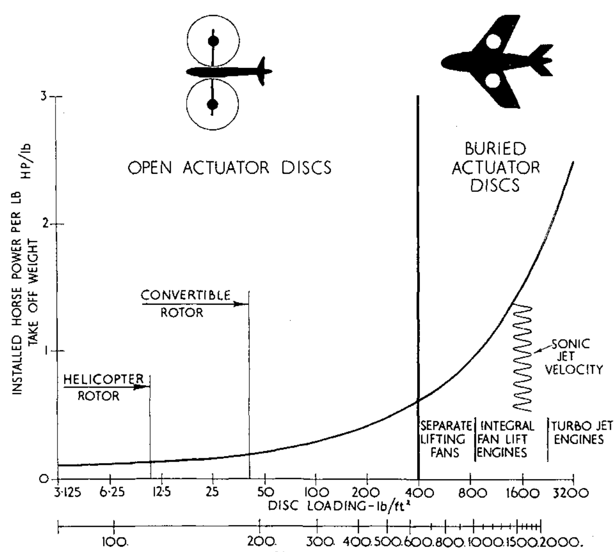

Fig. 1 Installed horsepower vs disk loading.

Figure 1 shows the rise of induced velocity and vertical takeoff power per unit thrust with increase of disk loading. It also gives the approximate ranges of disk loading for various typical thrust producers (helicopter rotors, convertible rotors, lifting fans, integral fan lift engines, and jet lift engines). A fundamental distinction is made between large open actuator disks and small actuator disks buried in wing or fuselage. Since the area of the latter is necessarily much less than that of the silhouette of the aircraft, it can be shown that, with typical wing loadings of about 100 psf, the loading of the buried disk must exceed 400 psf.

Noise

Noise is an unavoidable by-product of powered lift. "White noise" is caused by the vorticity in the slipstream, and, in addition, there are distinct frequency bands at high db levels produced by periodic phenomena in the thrust producer (rotating blades, intake noise, etc.). In Appendix A from data on helicopters and fan lift engines an estimate is made of the noise at various distances of a "standard" 100,000 lb VTOL aircraft as a function of disk loading. It is given in Fig. 2, which illustrates how steeply noise level rises with disk loading.

From investigations on acceptable noise levels in office and residential accommodation[11, 19] it is shown that in cities such VTOL aircraft, because of the noise they generate, will make certain areas unsuitable for this type of accommodation and that the affected area increases rapidly with disk loading (Fig. 3). It is evident that VTOL aircraft with disk loadings much above 25 psf are not suitable for city center operations.

"Balanced" VTOL Aircraft

Associated with the two modes of flight of the VTOL aircraft is the concept of two alternative assemblies, namely, 1) the cruise assembly that contains all those items (or parts thereof) that (with regard to the relevant specification) form an optimum cruise configuration together with the remainder of the aircraft, which in this case is the parasitic weight in the cruise mode W_{pc}; and, alternatively, 2) the hovering assembly that similarly corresponds to the most suitable hovering configuration and also leaves a remainder in the form of parasitic weight in the hovering mode W_{ph}.

It is evident that the more these assemblies can be eclipsed into one another, the smaller will be the parasitic weights W_{pc} and W_{ph} and, consequently, the more efficient will be the VTOL aircraft. How the eclipsing can best be achieved depends largely on the specified performance in the two modes of flight. Thus, for instance, if the duration required in the hover mode is long compared with that in the cruise mode, it is important to keep the hover power and the specific fuel consumption as low as possible. If, on the other hand, hovering is required for only a short period during takeoff and landing, then engine weight alone matters and fuel consumption in this mode is of little consequence.

Balanced Lift and Propulsion Weights

With the foregoing, the "balancing" of the VTOL aircraft design presents two distinct aspects. First, there is the employment of convertible means that function efficiently in both modes of flight. In this way a fuller eclipsing of the two flight assemblies is achieved, and parasitic weights are reduced. A good example of this is the convertible rotor, which in the hover mode acts as an adequate lifting means and in the cruise mode as an efficient propelling means. The same applies to the lift-thrust engine (vectored thrust). Another example would be the auxiliary wing of the compound helicopter, the major part of which in the hover mode serves as undercarriage structure, leaving only a small parasitic weight in this mode. Second, when convertible means are inappropriate, the selection of two separate means is pertinent, such that an equitable distribution of parasitic weights in the two assemblies is achieved, since too large a weight penalty in any one of the flight modes would have serious repercussions throughout; for instance, in a compound helicopter the optimum combination of rotor and fixed wing, or in a VTOL strike aircraft the optimum combination of thrust and lift means.

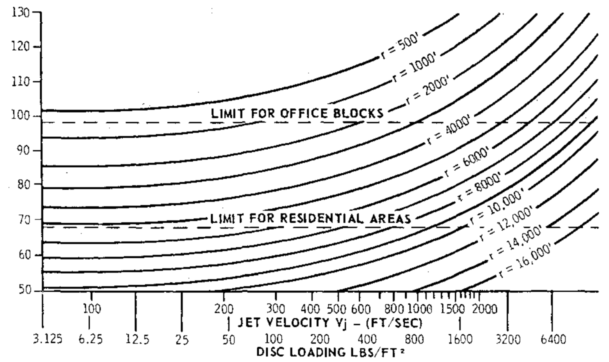

Fig. 2 Noise levels vs disk loading and distance.

It is on this aspect that careful design (optimization studies) can bring the greatest rewards. It is also here, we think, that some of the executed VTOL designs (tilt-wing and jet lift) fall demonstrably short of optimum performance capabilities.

Balanced Power

As powerplants and fuel constitute a major part of the weight of a VTOL aircraft, careful balancing of installed power is very important.[2] The power per pound takeoff weight required in the hover mode is shown in Fig. 1. It rises steeply with disk loading. The power required in the cruise mode can vary greatly. In Fig. 4 the cruise power for various types of aircraft (ranging from low speed to supersonic speed) is collated[15] as a function of cruise Mach number. This power can be translated into equivalent static sea-level power, or "installed power per pound takeoff weight" with proper allowance for the effects of cruise height and cruise speed. Now equating this installed power for the cruise mode with that for the hover mode (see Figs. 4 and 1) the correct, or "balanced," disk loading for the foregoing types of aircraft can be established. This is done in Fig. 5, which shows that for low subsonic speeds (200 knots SL) conventional helicopter disk loadings are most suitable. For economic operation at high subsonic speed (400 knots at 30,000 ft) the optimum disk loading is around 25 psf (typical

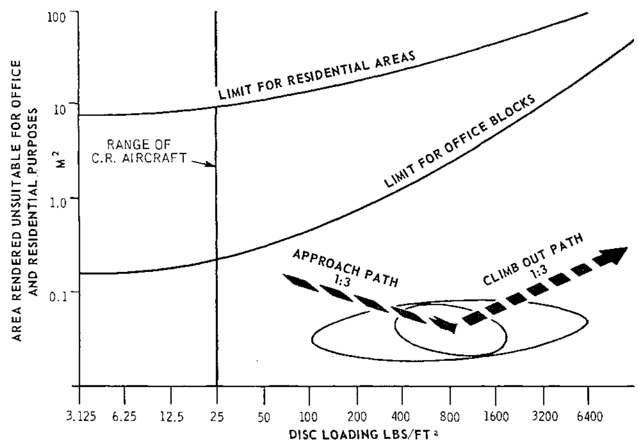

Fig. 3 Noise affected areas vs disk loading.

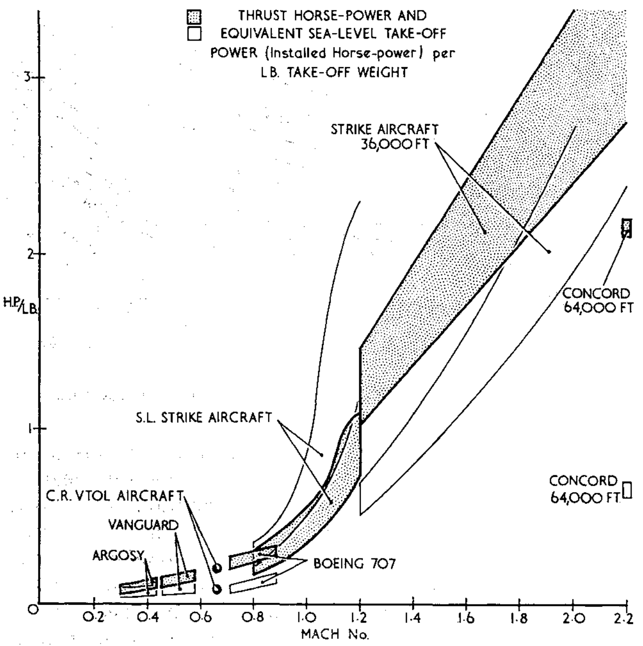

Fig. 4 Thrust horsepower and equivalent sea level takeoff power vs disc loading

convertible rotor). For supersonic aircraft very much higher disk loadings are appropriate (fan lift engines). There does not appear to be in this analysis any useful application for disk loadings of about 400 psf (separate lifting fans) which is evidently too high for efficient subsonic aircraft and, on the other hand, too low for supersonic aircraft. Such fans may perhaps find a use in transonic SL/strike aircraft.

Speed Limitations of the Helicopter

The performance of pure and compounded helicopters has been discussed in various papers,[10] and therefore only a brief recapitulation of the general conclusions is given here. The following cruise speeds may be achieved by the helicopter: pure helicopter, 130 knots; compound helicopter, 180 knots; and compound helicopter, 240 knots. Besides these speed limits there is the consideration of power demand and complexity, which leads to decrease in disposable load as we move from pure helicopter to compound helicopter.

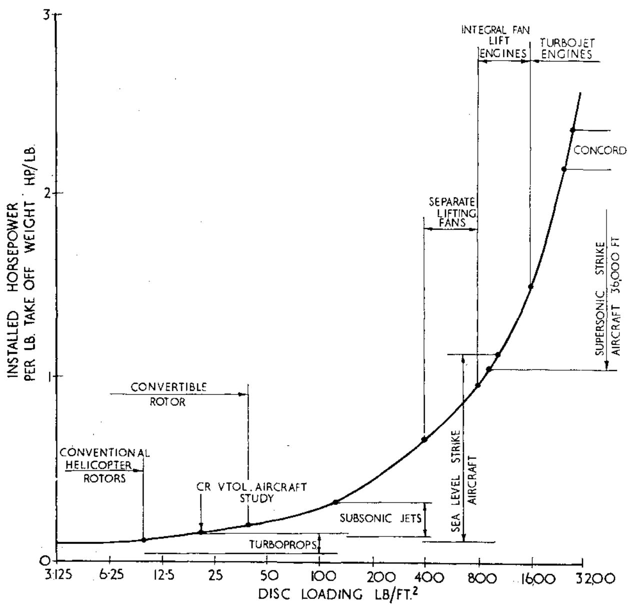

Fig. 5 Balanced VTOL solutions.

From Table 1 it can be seen that, with the helicopter, increased speed can be achieved up to a certain limit by accepting more structure weight and consequently less payload and, above all, increased complexity. Thus, in the absence of other VTOL competitors in this speed bracket, here is a choice between a slower, cheaper aircraft carrying more payload, and a faster, more expensive aircraft carrying less. In the studies with type 194, we have concluded that a moderately compounded tandem helicopter represents probably the best compromise for all-up weights (AUW) between 30,000 and 60,000 lb. Here, for only a small weight penalty, 50% of the rotor lift at design cruise speed is transferred onto a small fixed wing that also serves as the support for the undercarriage. That the moderately compounded helicopter offers the highest productivity factor is also the conclusion reached in Ref. 8.

The Convertible Rotor

The convertible rotor (CR) is designed to provide lift in the hover mode as well as propulsion in the cruise mode. As the requirements in the two modes differ fundamentally, it is difficult to meet them both efficiently by one means. A number of solutions to this problem have been attempted which apparently have little in common (Bell XV-3, Vertol 76, Hiller X-18, etc.).[3, 5, 7]

In the hover mode, the thrust produced is a maximum, being approximately equal to the takeoff weight of the aircraft. This, together with the rotor tip speed at hover, determines the blade area. If duration in this mode is limited to takeoff and landing only, then fuel consumed is of no consequence. It is, however, important that there be no parasitic weight from the power plant in either mode of flight, i.e., that the "installed power" be the same for both modes. As the convertible rotor is limited to subsonic operations, "installed power" is relatively low (see Fig. 5) and therefore the "balanced" disk loading is also low (of the order of 25 psf and less). The convertible rotor is thus a comparatively large device, more like a helicopter rotor than a conventional aircraft propeller. In order to accommodate such large rotors along the wing span of the aircraft the number is limited to two, one on each wing.

It is noteworthy that these two conclusions are at variance with some other proposals for tilt-wing aircraft[2, 5, 6] and also with some actual aircraft,[17] such as the X-18 and XC142. Their disk loadings are much too high for the reasons given, and we consider them therefore "unbalanced VTOL designs" carrying excessive parasitic weights.

The aerodynamics of the convertible rotor is rather complex owing to the different aerodynamic conditions of the annular elements of the rotor. The basic mechanism can, however, be recognized readily by considering only one representative annular element (say, at three-quarters rotor radius). Such an investigation, presented in Appendix B, shows the following interesting results.

The main dilemma in the CR springs from the great difference between the thrust at hover and that at cruise. There is prima facie less blade area required in cruise than at hover. If, however, variable blade area for reasons of mechanical

Table 1. Helicopter performance

Type	Design speed, knots	Basic weight	AUW (all-up weight)	Disposable load	Disposable load, AUW
S.61	134	11,094	18,788	7694	0.410
Vertol 107	141	10,425	18,450	8025	0.435
Rotodyne	200	40,945	60,654	19,709	0.325
Type 194	200	23,027	37,100	14,073	0.379
Type 194 with propeller	230	23,727	37,100	13,373	0.360

complexity is ruled out, there remain the parameters of blade speed, air density, and lift coefficient, in order to accommodate this difference in thrust.

It is clearly desirable to maintain a maximum of rotor tip speed in hover, but this may be desirable also in the cruise mode. The second parameter, air density, can be used to advantage; i.e., since takeoff and landing generally take place near sea level, an effective air density differential can be achieved by cruising at substantial height (30,000–35,000 ft). The third parameter, the blade lift coefficient, can be optimized by selecting a high C_{L_h} in the hover mode and accepting a rather lower C_{L_c} in the cruise mode, such that these two coefficients lie on either side of $C_{L\mathrm{opt}}(L/D = \text{maximum})$. A high rotor tip speed in hover, together with a high C_L is thus desirable ($C_{L\mathrm{crit}} V_{T\mathrm{crit}}{}^2 = \text{maximum}$). This may lead to airfoils specially designed for this purpose. Such compromise is not needed in the design of conventional propellers, which generally have less profile drag in cruise than the convertible rotor (which suffers from excessive blade area). Fortunately, however, there is in the CR a redeeming feature in the very large disk area, which reduces Froude losses to nearly zero. It is shown in Appendix B that the cruise efficiency of an airscrew can be expressed by

$$\eta = 1 - \lambda m/[1 + (m/\lambda)]$$

where $m = \epsilon_c + \iota_c$. Thus, the efficiency, apart from the advance ratio λ depends solely on the sum of $\epsilon_c + \iota_c$. Although in the conventional propeller ϵ_c can be chosen to be near a minimum, ι_c is generally rather high because of diameter limitations. In the CR, by contrast, ϵ_c is higher, but ι_c is close to zero (see Fig. 6). The CR can, therefore, contrary to general notion, achieve equal, or even higher, cruise efficiencies than the conventional propeller, especially if σ (ratio between air density at cruise and air density in hover) is well below 1.

The analysis also shows that with conventional propellers $\eta_{\max}$ is obtained for values of $\lambda < 1.0$. This is mainly because for a given thrust, blade lift and, therefore, ι_c increases with λ. In the CR, however, ι_c is such a small term that changes thereof are of no consequence. ϵ_c on the other hand (at these low values of C_L) decreases with increase of λ tending to improve over-all efficiency. Figure 7 shows that $\eta_{\max}$ is reached at a λ of about 1.4. Thus, for a given helical tip speed, the economic cruise speed of the CR is higher than that of a conventional propeller. Conversely, this high optimum λ encourages a reduction in cruise tip speed. Considering, moreover, that because of lower C_L values in cruise, the CR's helical tip speed (critical Mach-number) itself is higher, a substantial (about 15%) gain in cruise speed has been achieved. Whereas conventional propeller-driven aircraft are limited to about 350 knots the CR VTOL aircraft can cruise efficiently up to speeds of about 400 knots. This has a direct bearing on operating cost.

Another significant feature emerging from the analysis is a substantial difference between the rotor speed at hover $R\omega_h$ and that at cruise $R_{\omega c}$, the latter being about two-thirds the former. This speed difference is fundamental and must be taken care of in the design of the powerplant for this type of aircraft. It points to the use of a free shaft turbine, where the speed of the gas generator is independent of that of the power turbine, which is variable over the range indicated.

One important parameter in the design of the CR which cannot be obtained from the simple annular element analysis in Appendix B, is the blade twist. In the complete analysis, where the rotor performance is obtained by integrating the behavior of the annular elements, it is evident that radial lift distribution in hover as well as in cruise must be considered carefully in order to achieve good performance in both modes of flight. For any given design this produces two optimum blade twists, one for hover and the other for cruise. This is shown in Fig. 8, which also gives the difference between hover and cruise twist. It is noteworthy that this difference

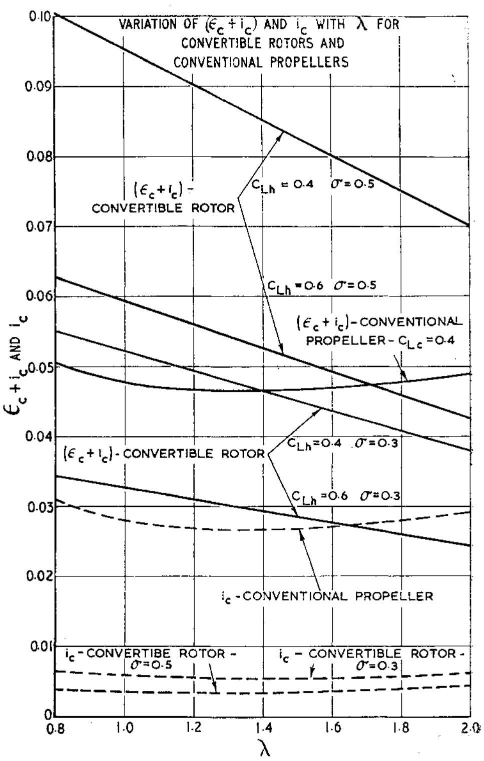

Fig. 6 Blade profile and induced loss factors vs. axial advance ratio.

diminishes as cruise λ increases beyond the value of 1. There remains, nevertheless, a significant difference (up to 6°) which indicates a variable geometry rotor in terms of twist. This may be essential, because small errors in blade twist will cause large changes in radial thrust grading which affects efficiency, especially in the cruise mode where both induced angles and angles of attack are very small. Moderately variable blade twist, we believe, can be accomplished without undue complexity.

Convertible Rotor Studies

Since 1953 the author has been concerned with several feasibility studies, which in their sequence reveal a metamorphosis of concept right up to the present standard. We discovered that only when the disk loading is comparatively

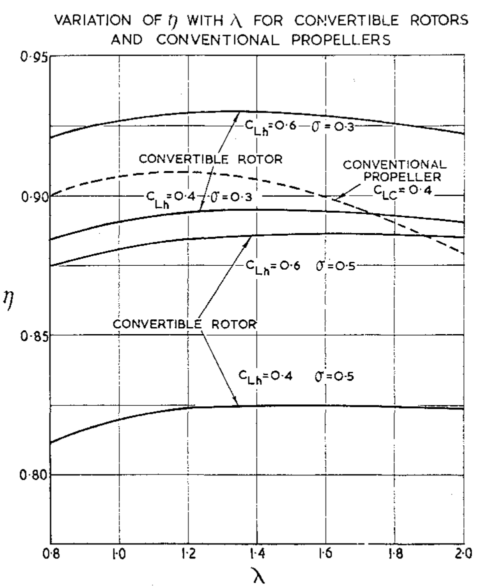

Fig. 7 Rotor efficiency vs. axial advance ratio.

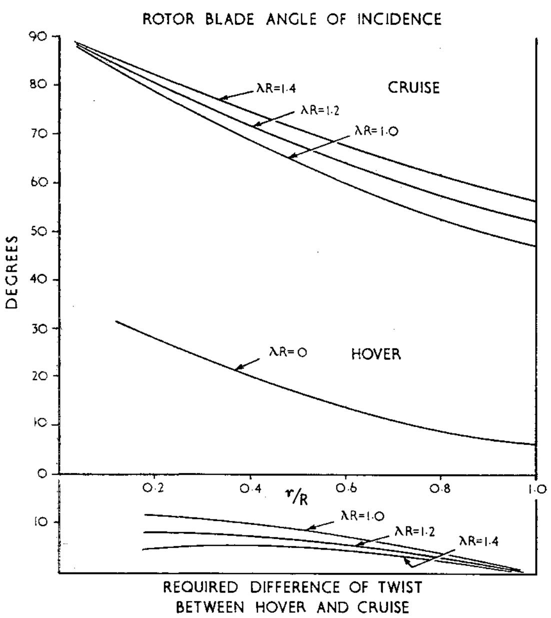

Fig. 8 Rotor blade, angle and twist.

low is it possible to achieve high efficiency both in hover and cruise and that the disadvantages of the large rotor (by comparison with the conventional propeller) are more than outweighed by its advantages. This led to the two recent studies, both on configurations with twin rotors of large diameter. The object of these studies was to "transform" a conventional fixed-wing aircraft into a CR VTOL aircraft of approximately the same AUW with the minimum change in layout in order to obtain a good basis for performance comparison. The parts affected in this "transformation" were mainly propellers, reduction gears, engines, wings, tail plane, and undercarriage. The fuselage with cockpit and passenger cabin including furnishings systems, equipment, etc., which represent a large proportion of the aircraft's weight, remained unchanged. This procedure insured that uncertainties in the weight estimate were confined to the "transformed" area, and even if large errors were made here the over-all error remained relatively small. The starting point in the first study[16] is the Vickers Viscount, which was transformed into a military CR VTOL transport (Fig. 9), and in the second study,[21] the Vickers Vanguard, which was transformed into a civil CR VTOL transport (Fig. 10).

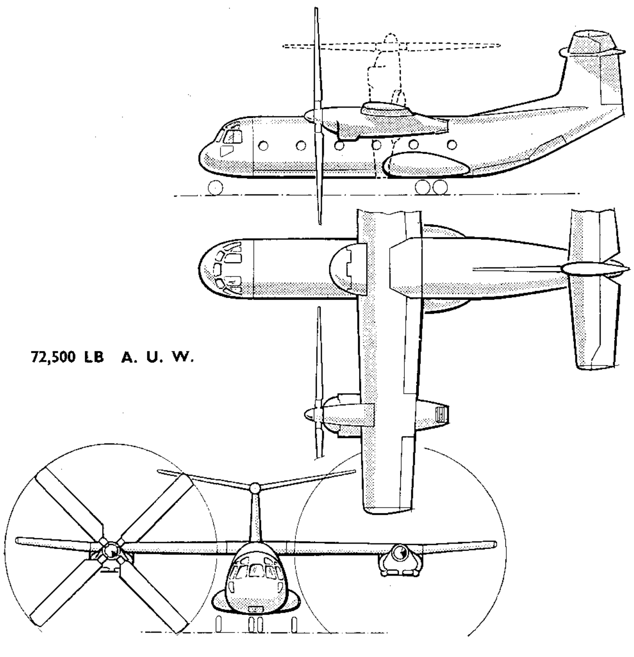

Fig. 9 Convertible rotor VTOL military transport aircraft.

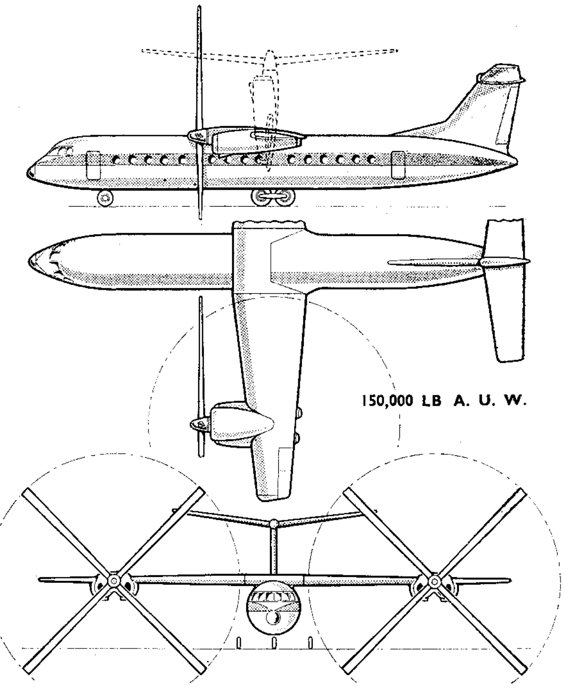

Fig. 10 Convertible rotor VTOL civil transport aircraft.

These studies produced a number of interesting points, in particular the following.

1) Cruise economy of the CR VTOL aircraft is as good as that of the fixed-wing aircraft, especially at heights above 25,000 ft.

2) The optimum "balanced" disk loading is relatively low (about 24 psf).

3) The propulsive efficiency of the large rotor is very high, even at extreme advance ratios. This permits the CR VTOL aircraft to cruise about 50 knots faster than the turboprop aircraft.

4) For a given all-up weight W the weight of the transmission system W_T, within the limits examined, remains sensibly independent of rotor radius R. Briefly, the reason for this is as follows: The weight of the gearboxes, which constitute by far the largest part of the transmission system, varies directly[13] with output torque T. The major part of the takeoff power P, which determines the transmission, is induced power P_i. Thus

$$P \simeq P_i = W\left(\frac{P}{2\rho}\right)^{1/2} = \frac{1}{R}\left(\frac{W^3}{2\rho\pi}\right)^{1/2}$$

as the tip speed V_T of the rotor remains sensibly constant, the rotor speed varies as $\omega = V_T/R$. Thus the torque $T = P/\omega \simeq (W^3/2\rho\pi)^{1/2}/V_T$ which is independent of R. Consequently, W_T does not vary with R.

5) For a given AUW the weight of rotor blades increases with rotor radius.[1, 9] This fact must be accounted for in balancing the disk loading.

6) The weight penalty, from which the CR VTOL aircraft suffers by comparison with the conventional turboprop aircraft (Fig. 11) is mainly in the rotors and the transmission system. From the airworthiness point of view these parts are critical, at least in the takeoff and landing phase, and must, therefore, be equipped with appropriate "fail safe" devices[22] and carry adequate safety factors. The other reason for the high weight is the comparatively high rotor torque. Figure 11 also shows that the weight available for cruise fuel and payload is 38 and 25% of AUW (for fixed-wing

Table 2 Operating parameters of various transport aircraft

Aircraft or design study	Type	AUW	Cruise Speed, knots	Cruise Height, ft	Terminal losses, min Flying time	Terminal losses, min Ground time
S.61	Pure helicopter	18,800	120	2,000	6	30
Rotodyne Z	Compound helicopter	60,650	200	5,000	6	30
Type 194 C	Compound helicopter	37,100	200	5,000	6	30
CR VTOL study	CR VTOL aircraft	150,000	400	30,000	12	30
V. Vanguard	Fixed-wing turboprop	146,500	350	25,000	20	105
Boeing 707	Fixed-wing turbojet	327,000	500	35,000	20	105
SST	Supersonic jet	275,000	1200	75,000	30	105

aircraft and CR VTOL aircraft, respectively). Thus the net weight penalty of the CR VTOL aircraft is 13% of the all-up weight.

7) A brief study comparing the CR VTOL aircraft with the deflected slipstream VTOL aircraft has shown that in balanced designs the leading rotor parameters are the same in both configurations and that the main difference is the deflection of the slipstream by the wing as against the rotation of the rotor (and wing). Both schemes are equally efficient in cruise, but the latter is aerodynamically superior in the hover (losses due to deflection are about 15%). With regard to weight penalties there is little to choose between the two arrangements.

These findings seem to be in agreement with those of other investigators.[4]

Comparisons

Now we shall compare operating costs and journey times between city centers of the types of VTOL aircraft discussed and of some typical fixed-wing aircraft. They are given in Table 2, together with leading particulars.

The direct operating costs (DOC) have been calculated by the method in Ref. 13 and from data obtained from various sources. The fuel reserves assumed are 1) helicopters, 1-hr holding; 2) fixed-wing aircraft, 1-hr holding plus 175 miles diversion; and 3) CR VTOL aircraft, $\frac{2}{3}$ fixed-wing reserves.

Terminal flying time losses are given in Table 2. They consist of airfield allowances and losses in climb and descent. The helicopter has the smallest time loss. The CR VTOL aircraft has the airfield allowance of the helicopter but loses more time in climbing to its cruise height. The fixed-wing aircraft has still higher time losses, both on the airfield and in climb. The terminal time loss of the supersonic transport (SST) is, to a large extent, determined by the time needed to climb at subsonic speed to about 40,000 ft (the minimum height for transition to supersonic speed), and from there to cruise height. It has been calculated from a typical flight plan given in Ref. 20.

In Fig. 12, DOC's per seat mile are given for these aircraft as a function of stage length. The helicopter suffers in this comparison from scale effect that favors the larger aircraft, but even so it is obvious that the helicopter can only achieve reasonably low DOC's at very short stage lengths because of its lack of real speed, and its design should, therefore, be optimized for such journeys. The fixed-wing aircraft achieves very low DOC minima because of its high cruise speed together with its relatively large payload. Its efficiency, however, deteriorates as the stage lengths decrease mainly because of its large terminal flying time losses. The latter applies particularly to the SST. The CR VTOL aircraft enjoys the high cruise speed of the fixed-wing aircraft together with the small flying time losses of the helicopter. Its percentage payload, however, is somewhat reduced due to the weight penalties in the rotor system and the power plant. Indirect operating costs are given in Fig. 13.

In the evaluation of journey times, because of the difference between VTOL operations and others, different ground time losses have been assumed. With this and other assumptions in Table 2 total journey times between city centers have been computed for stage lengths of 200 and 800 naut miles. They are given in Fig. 14. The 200-naut-mile

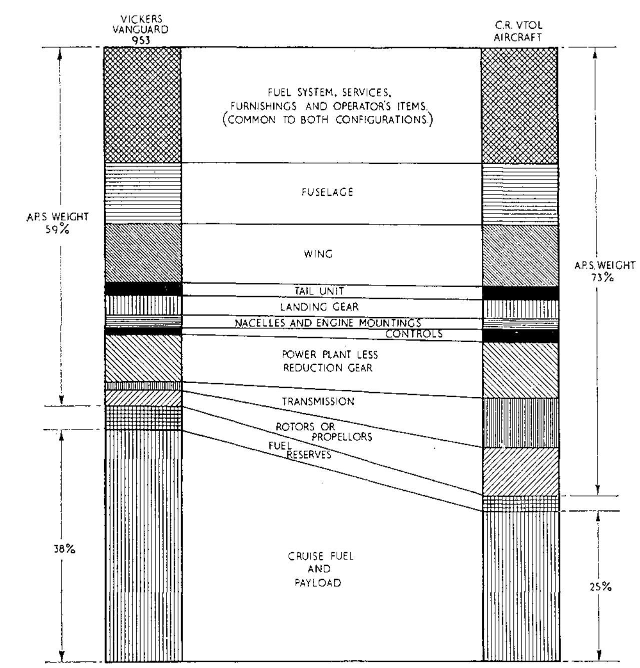

Fig. 11 Weight analysis for CTOL and CR VTOL aircraft.

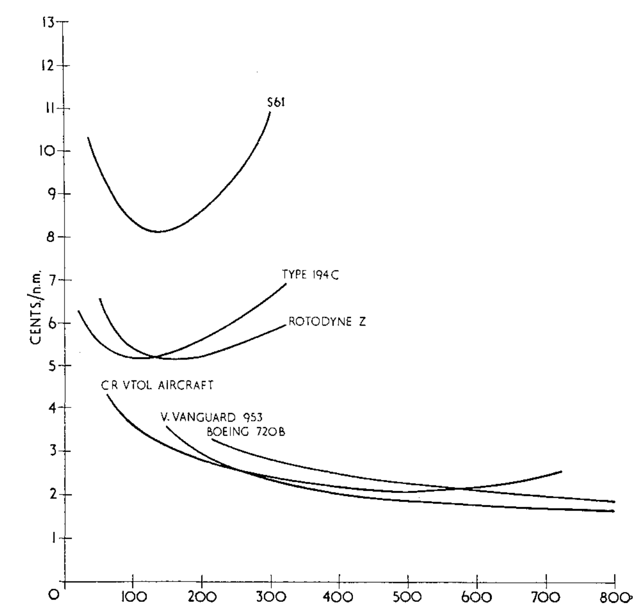

Fig. 12 Direct operating costs vs stage lengths.

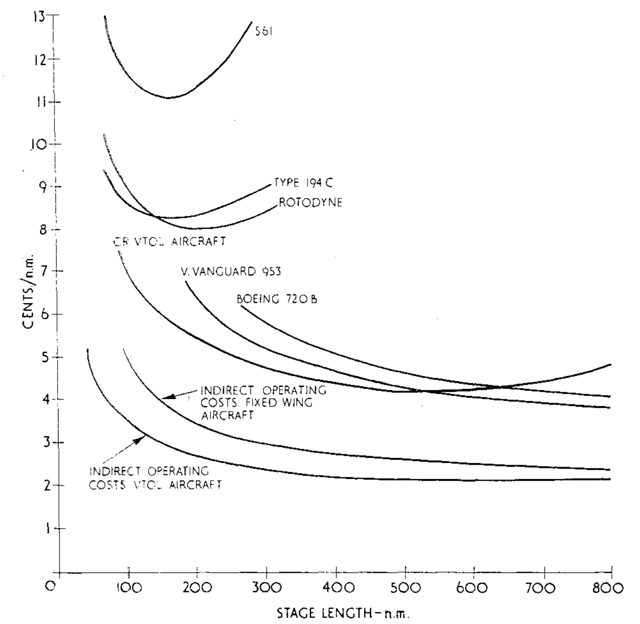

Fig. 13 Indirect operating costs and total costs vs stage lengths.

journey takes a little over one hour with the CR VTOL air craft and more than twice as long with fixed-wing aircraft. The helicopter lies about half-way between these two. The 800-mile journey takes $3\frac{3}{4}$ hr with the CR VTOL and just a little more with the SST. The turbojet passenger arrives about 1 hr later and the turboprop passenger $1\frac{1}{2}$ hr later. Thus, the CR VTOL aircraft offers the highest speed of transportation between city centers up to a distance of 800 miles.

Conclusions

1) In order to avoid excessive weight penalties in the two modes of flight, which are detrimental to efficiency, a VTOL aircraft must be "balanced."

2) Balanced VTOL solutions are feasible in a wide spectrum of disk loadings, depending mainly on the requirements in the cruise mode. Whereas for supersonic cruise speeds disk loadings of over 1000 psf (fan lift engines) are most suitable, for economic operations at high subsonic cruise speed the optimum disk loading is about 24 psf, for which a CR is the correct solution.

3) The noise generated by VTOL aircraft is, apart from all-up weight, mainly a function of disk loading. Operation of VTOL aircraft in city centers is not compatible with criteria for acceptable noise levels (dormitories and business centers) if the disk loadings of these aircraft are much more than 25 psf.

4) The large diameter CR has a surprisingly high efficiency in the cruise mode, despite its excessive blade area, and can at substantial cruise height surpass that of the conventional propeller. Its maximum efficiency, moreover, lies at an advance ratio which is substantially higher than that of the conventional propeller. This permits CR aircraft to cruise efficiently at higher speeds than the conventional propeller.

5) The net weight penalty in the CR aircraft, by comparison with the fixed-wing aircraft (mainly due to the increase in the weight of rotor, transmission and powerplant), is about 13% of the all-up weight.

6) The CR VTOL aircraft offers lower operating costs than any other type of aircraft for stage lengths up to about 500 naut miles. It offers the fastest means of transportation between city centers for distances of up to 800 naut miles.

7) The object of this paper, to indicate the domain of the convertible rotor, has been achieved by investigations made into certain economic and technical aspects. For a comprehensive engineering investigation, however, a much wider area needs to be explored. Despite the evidence of substantial efforts in this field, especially in the United States, we have yet some way to go to a commercially acceptable solution. We are fully aware of the many problems still unsolved but have reason to believe that they are capable of solution and that the promise of this paper is not merely academic. If this proves true, then the outmoded propeller may in time experience a profound reincarnation in the shape of the convertible rotor.

Appendix A: VTOL Noise in Builtup Areas

It is the object of this analysis to estimate noise levels of VTOL transport aircraft and to assess their chance of operating in builtup areas without causing nuisance. There is a good deal of information on noise from helicopters, which is collated in Fig. 15. The dominating noise here is intake noise, low frequency noise from tail rotors, etc., and the contribution from jet noise is small. The noise of high disk loading VTOL aircraft, on the other hand, springs predominantly from the high-velocity jet stream. Howell[20] and Denning[14] have made estimates, which are also shown in Fig. 14. In order to bring these data to a common denominator we have considered a "standard" VTOL aircraft of 100,000 lb all-up weight at a distance of 500 ft. With the assumption of +3 PNdB per doubling of AUW and −6 PNdB per doubling of distance and an additional allowance for molecular absorption of sound energy of −3PNdB for each 1000 ft, the convex curve in Fig. 15 is obtained, as well as the family of curves in Fig. 2 giving the noise level of the "standard" VTOL aircraft at various distances as a function of disc loading. From various reports[6, 11, 19] two important noise level criteria have been obtained, namely: daytime acceptable noise level in offices (speech interference criterion), 65 PNdB; and night-time acceptable noise level in residential areas (sleep interference criterion), 45 PNdB.

It is further assumed that for office accommodation in this critical area with double windows and air conditioning the noise attenuation is 25 PNdB. In residential buildings with single windows, etc., the noise attenuation is only 15 PNdB. There is an additional attenuation in builtup areas because of shielding that is assessed at 8 PNdB. Thus the acceptable

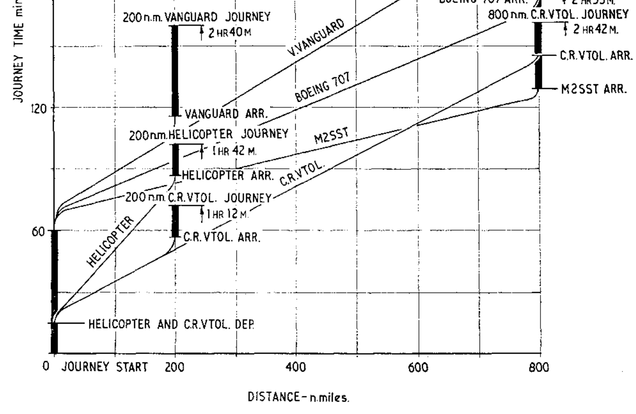

Fig. 14 Journey times vs distance.

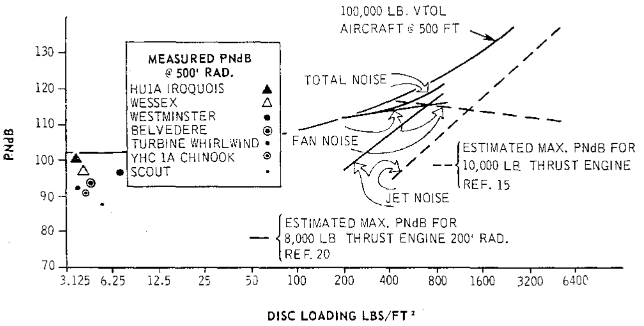

Fig. 15 Noise levels vs disk loading.

noise level above the roof of the building is, in the city business center, 98 PNdB, and in the suburb residential area (night), 68 PNdB.

These noise criteria are shown in Fig. 2 as two horizontal lines. The points of intersection between these lines and the family of noise level curves represents the minimum permissible distance r between the standard CTOL aircraft during takeoff and landing and the locus of residential and business accommodation, respectively. This distance depends primarily on disk loading.

It is further assumed that the operation of VTOL aircraft in builtup areas consists basically of two flight paths (approach and climb out) inclined at an angle of about 1 in 3 (Fig. 3). It is possible to draw lines connecting points on the ground of equal noise level (isobells), and it can be shown that such lines have the form of concentric ellipses with $2r$ as the minor axis and $6r$ as the major axis where r is given in Fig. 2. The area within such an ellipse is $3\pi r^2$. This is given in Fig. 3 as a function of disk loading. It represents the area that is rendered unsuitable for residential or office accommodation because of the noise from the VTOL activities.

Appendix B: Performance of the Convertible Rotor

As the performance of a rotor or propeller is computed by integrating the performance of it's elements, considerable mathematical complexity is involved which to some extent hides the basic aerodynamic mechanism. The annulus at about three-quarters rotor radius is fairly representative of the behavior of the whole rotor or propeller, and it can be examined analytically. Consider a convertible rotor in the takeoff mode. Let

R	=	rotor radius
$r = \frac{3}{4}R$	=	radius of typical annulus
$2\pi r dr$	=	annular area dA
$Vm = 850$ fps	=	critical velocity of blade tip
c	=	blade chord
n	=	number of blades
dT_h	=	annular thrust
$dT_h/dA = p_h$	=	annular loading
$V_{ih} = (p_h/2\rho_h)^{1/2}$	=	induced velocity
ω_h	=	angular velocity
subscript 0	=	profile conditions
subscript h	=	hovering conditions

Since the disk loadings proposed for convertible rotors are less than 25 lb/ft², the induced velocity V_{ih} is small compared to blade tip speed; thus, the induced angle i_h is also small (see Fig. 16). Therefore,

$$dL_h \simeq dT_h \qquad Vr_h \simeq r\omega_h$$

$$\begin{aligned} dT_h &= dL_h = CL_h \cdot \rho/2 \cdot ncVr_h^2 dr \\ &= p_h \cdot 2\pi \cdot r dr \end{aligned} \tag{B1}$$

$$nc = \frac{2\pi r p_h}{C_{Lh}\rho_2(\frac{3}{4}Vm)^2}$$

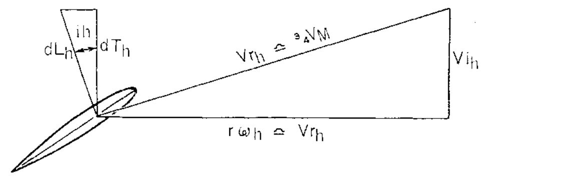

Fig. 16 Rotor blade geometry in hover.

It will be seen later that the high thrust required in the hover mode necessitates high values for C_{Lh} as well as for the maximum tip speed (Vm). For this reason ($\frac{3}{4}Vm$) has been substituted for Vr_h in (B1), where the subscript indicates critical Mach number.

The airfoil selected for this examination is NASA 16209 with an addition of $\Delta C_D = 0.002$ for contingency. Its drag characteristic can be expressed by the equation

$$C_D = k - lC_L + mC_L^2 \tag{B2}$$

where k, l, and m are positive constants, namely, $k = 0.0087$, $l = 0.025$, and $m = 0.058$. Assuming further, blade radius $R = 32$ ft, disk loading $p_h = 25$ psf, and $C_{Lh} = 0.60$. This gives C_{D0h} equal to 0.0147, with $\rho_h = 0.00238$ and $nc = 12.9$ ft.

The profile power is $dP_0 = C_{D0h} \cdot \rho_h/2 \cdot ncV_{rh}^3 \cdot dr$; and the profile power per pound of thrust is given by

$$\frac{dP_0}{dT_h} = \frac{CD_{0h}}{CL_h} \cdot Vr_h = \frac{0.0147}{0.60} \cdot 640 = 15.7 \;\frac{\text{lb-ft}}{\text{sec}} \Big/ \text{lb}$$

$$\text{induced power per pound of thrust} = W\left(\frac{p_h}{2\rho_h}\right)^{1/2} \times$$

$$\frac{1}{W} = V_{ih} = \left(\frac{25.0 \times 421}{2}\right)^{1/2} = 72.5 \;\frac{\text{lb-ft}}{\text{sec}} \Big/ \text{lb}$$

Then the total hovering power per pound of thrust is 88.2 (lb-ft/sec)/lb. The major part of this total is induced power. This total corresponds to a power loading of 6.24 lb/hp.

Cruise Mode

The relevant velocities, angles, and forces are shown in Fig. 17. Subscript c refers to cruise conditions. The resultant speed Vr_c is the sum of the speed vectors of $r\omega_c$ and $\lambda r\omega_c$. The induced speed vector Vi_c does not alter materially the magnitude of the resultant speed Vr_c but only its direction. Now, $\tan\theta = \lambda$, and $\tan i_c = Vi_c/Vr_c$. The D/L ratio of the airfoil $= dD/dL = \epsilon_c$. Assuming that i and ϵ are small angles, and $\tan i_c = i_c <<< 1$, and $\tan \epsilon_c = <<< 1$, it can be shown (see Fig. 17) that

$$\tan(\theta + i_c) = \frac{\lambda + i_c}{1 - \lambda i_c}$$

$$\tan\phi = \frac{dT_c}{dQ_c} = \frac{(1 - \epsilon_c i_c) - \lambda(\epsilon_c + i_c)}{\lambda(1 - \epsilon_c i_c) + (\epsilon_c + i_c)}$$

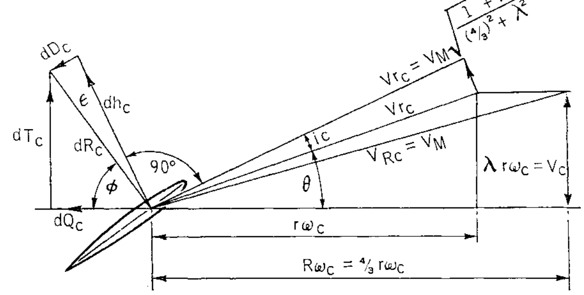

Fig. 17 Rotor blade geometry in cruise.

As from the foregoing, $\epsilon_c i_c$ can be neglected:

$$\tan\phi = \frac{dT_c}{dQ_c} = \frac{1 - \lambda(\epsilon_c + i_c)}{\lambda + (\epsilon_c + i_c)}$$

Propeller Efficiency:

$$\eta = \frac{V_c dT_c}{r\omega_c dQ_c}$$

$$= \lambda\left[\frac{1 - \lambda(\epsilon_c + i_c)}{\lambda r(\epsilon_i + i_c)}\right] \tag{B3}$$

$$= \frac{1 - \lambda(\epsilon_c + i_c)}{1r(1/\lambda)(\epsilon_c + i_c)}$$

Where $(\epsilon_c r i_c)$ vanishes, $\eta = 1$ for any value of λ. However for any *finite* value of $(\epsilon_c + i_c)$, $\eta < 1$ and $\eta_{\max}$ can be found by differentiating Eq. (B3) with respect to λ which for small values of $(\epsilon_c + i_c)$ gives

$$\lambda_{\text{opt}} \simeq 1 - (\epsilon_c + i_c)$$

It is also clear that η depends only on λ and the sum of ϵ_c and i_c. This means that ϵ_c can be increased at the expense of i_c without affecting the efficiency of the rotor element. Now

$$Vr_c = r\omega_c(1 + \lambda^2)^{1/2} \tag{B4}$$

$$\frac{\omega_c}{\omega_h} = \frac{1}{[1 + (\frac{3}{4}\lambda)^2]^{1/2}} \tag{B5}$$

$$dR_c \simeq d_{Lc} = C_{Lc}\cdot\rho_2\cdot ncVr_c^2 dr$$

$$= C_{Lc}\rho_2\cdot nc(r\omega_c)^2 dr\cdot(1 + \lambda^2)$$

$$dT_c = dR_c\cdot\sin\phi = \frac{dR_c}{[1 + (1/\tan^2\phi)]^{1/2}}$$

$$= dR_c\frac{1 - \lambda(\epsilon_c + i_c)}{(1 + \lambda^2)^{1/2}}$$

neglecting $(\epsilon_c + i_c)^2$. Therefore

$$dT_c = C_{Lc}\cdot\rho_c/2\cdot nc(r\omega_c)^2 dr[1 - \lambda(\epsilon_c + i_c)](1 + \lambda^2)^{1/2} \tag{B6}$$

The mass flow per second passing through the element dA is $dM \simeq V_c\cdot dA\rho_c$. From momentum considerations, $dL_c = V_c dA\rho_c 2V_{ic}$ (where $2V_{ic}$ is the fully developed induced velocity in the vena contracta). Therefore

$$Vi_c = \frac{dL_c}{2\pi r dr}\cdot\frac{1}{2\rho_c\lambda r\omega_c}$$

$$= \frac{C_{Lc}nc}{8\pi r}\cdot\frac{Vm}{\lambda}\cdot\frac{1 + \lambda^2}{[(\frac{4}{3})^2 + \lambda^2]^{1/2}} \tag{B7}$$

$$i_c = \frac{Vi_c}{Vr_c} = \frac{C_{Lc}}{8\pi}\cdot\frac{nc}{r}\cdot\frac{(1 + \lambda^2)^{1/2}}{\lambda}$$

From Eq. (B3), $\epsilon_c = \dfrac{C_{Dc}}{C_{Lc}} = \dfrac{k}{C_{Lc}} - l + mC_{Lc}$ (B8)

Therefore,

$$\epsilon_c + i_c = \frac{k}{C_{Lc}} - l + C_{Lc}\left[m + \frac{1}{8\pi}\cdot\frac{nc}{r}\cdot\frac{(1 + \lambda^2)^{1/2}}{\lambda}\right] \tag{B9}$$

If the drag-to-lift ratio in cruising flight is ξ, then

$$\xi = \frac{dT_c}{dT_h} = \frac{C_{Lc}\cdot\rho_c/2\cdot nc(r\omega_c)^2 dr[1 - \lambda(\epsilon_c + i_c)](1 + \lambda^2)^{1/2}}{C_{Lh}\cdot\rho_{h/2}\cdot nc(r\omega_h)^2 dr}$$

with

$$\frac{\rho_c}{\rho_h} = \sigma$$

$$(r\omega_c)^2 = \frac{V_m^2}{(\frac{4}{3})^2 + \lambda^2}$$

$$\frac{(r\omega_0)^2}{(r\omega_h)^2} = \frac{1}{1 + (\frac{3}{4})\lambda)^2}$$

Then

$$C_{Lc} = C_{Lh}\left[\frac{\xi}{\sigma}\cdot\frac{1 + (\frac{3}{4}\lambda)^2}{(1 + \lambda^2)^{1/2}}\right]\left[\frac{1}{1 - \lambda(\epsilon_c + i_c)}\right] \tag{B10}$$

where $\lambda(\epsilon_c + i_c)$ can be neglected. If we examine a family of conventional propellers, C_{Lc} is usually chosen to give a good L/D ratio for the airfoil (for NASA 16209). This is approximately $C_{Lc} = 0.40$.

In this family, λ is varied while the thrust is kept constant with a view to obtaining the optimum λ to give $\eta_{\max}$. From (B10) and using NASA 16209 with $C_{Lc} = 0.40$,

$$\epsilon_c + i_c = \underbrace{\frac{0.0087}{0.40} - 0.025 + 0.4\Big[0.058}_{\epsilon_c} + \underbrace{0.04\frac{nc}{r}\frac{(1 + \lambda^2)^{1/2}}{\lambda}\Big]}_{i_c}$$

where c/r corresponds to the activity factor (AF) used in propeller design (AF $\simeq 780\ c/r$). In a four-bladed propeller with AF = 125, $nc/r = 0.64$.

Then with T_c, C_{Lc}, Vm, and ρ_c constant, nc/r must increase with increasing λ to maintain optimum efficiency. This can be seen from (B6) as

$$ncdr(r\omega_c)^2[1 - \lambda(\epsilon_c + i_c)](1 + \lambda^2)^{1/2} = \text{const}$$

where $\lambda(\epsilon_c + i_c)$ can be neglected, since it is small compared with 1. Also

$$(r\omega_c)^2 = \frac{V_m^2}{(\frac{4}{3})^2 + \lambda^2}$$

Therefore nc varies as $[(\frac{4}{3})^2 + \lambda^2/(1 + \lambda^2)^{1/2}]$, and therefore

$$\epsilon_c + i_c = \frac{0.0087}{0.4} - 0.025 + 0.4\left[0.058 + 0.04\left(0.64\frac{(\frac{4}{3})^2 + \lambda^2}{\lambda}\right)\right]$$

$$= 0.02 + 0.01\left[\frac{(\frac{4}{3})^2 + \lambda^2}{\lambda}\right]$$

These optimum values of $(\epsilon + i_c)$ and η are shown plotted against λ in Figs. 6 and 7. In the convertible rotor C_{Lc} is determined by (B10) and with (B9) gives

$$(\epsilon_c + i_c) = \frac{k}{C_{Lh}}\frac{\sigma}{\xi}\cdot\frac{(1 + \lambda^2)^{1/2}}{1 + (\frac{3}{4}\lambda)^2} + mC_{Lh}\left[\frac{\xi}{\sigma}\cdot\frac{1 + (\frac{3}{4}\lambda)^2}{(1 + \lambda^2)^{1/2}}\right] - l + \frac{\xi}{\sigma}\cdot\frac{p_h}{2\rho_h(\frac{3}{4}Vm)^2}\cdot\frac{1 + (\frac{3}{4}\lambda)^2}{\lambda}$$

if

$$\frac{\xi}{\sigma} = \tau \qquad \frac{1 + (\frac{3}{4}\lambda)^2}{(1 + \lambda^2)^{1/2}} = \chi \qquad \frac{p_h}{2\rho_h(\frac{3}{4}Vm)^2} = \Delta.$$

Table 3 Rotor performance vs λ

λ	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\epsilon_c + i_c$	0.0501	0.0477	0.0468	0.0466	0.0472	0.0479	0.0488
η	0.901	0.910	0.909	0.903	0.900	0.89	0.88

Then

$$(\epsilon_c + i_c) = \frac{k}{C_{Lh}} \frac{1}{\tau\chi} + mC_{Lh}\cdot\tau\chi - l + \tau\Delta\chi \frac{(1+\lambda^2)^{1/2}}{\lambda}$$

It is desirable that this expression be as small as possible. It is clear that k and m should be as small as possible whereas l should be large. This only expresses the characteristics of a good airfoil (low C_D for all values of C_L).

A low value of Δ is obtained by low disk loading and high tip speed. C_{Lh} is determined in the hovering mode to the extent that efficiency in hovering is very important (which means that C_{Lh}/C_{Dh} to be a near maximum). On the other hand efficiency in the cruise mode is also important, i.e., C_{Lc}/C_{Dc} should be a near maximum.

The relation between C_{Lh} and C_{Lc} is given by Eq. (B10). The factor which tends to reduce C_{Lc} (and thus worsen C_{Lc}/C_{Dc}) is the D/L of the aircraft. This effect is offset by σ if the cruising altitude is substantial ($\sigma = 0.3$ at 30,000 ft). C_{Lc} is also affected by the helix angle, i.e., λ and high λ can increase C_{Lc}/C_{Lh}.

It is clear however that C_{Lc} must be much less than C_{Lh}, and therefore it is not possible to obtain a good CL/CD ratio in both regimes of flight. This is the crucial problem in the convertible rotor. One is forced to select a fairly high C_{Lh} (above the optimum), limited in fact only by stalling and compressibility considerations, so that in the cruise regime C_{Lc} is not too low for efficient operations.

A numerical example based on the following assumptions has been calculated:

$$\xi = \tfrac{1}{12} \qquad \sigma = 0.3 \text{ and } 0.5 \qquad \rho_h = 0.002378$$

$$Vm = 850 \text{ fps} \qquad p_h\ 25 \text{ lb/ft}^2 \qquad C_{Lh} = 0.4 \text{ and } 0.6$$

$(\epsilon_c + i_c)$ and η are shown plotted against λ in Figs. 6 and 7.

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